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A Parametric Study of Functionally Graded Rotating Annular Fin

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Abstract

Performance analysis of the rectangular annular fins made of functionally graded material subjected to rotation is reported in the present work. The governing differential equation for the fins has been derived to study the temperature distribution of the fins with insulated tip. A parametric study is then carried out by varying grading parameters along with the parameter of dimensionless rotational speed of the shaft in the governing equation to investigate the effect of on fin performance. The results are presented in graphical form. The formulation is validated with benchmark results and good agreement is observed.

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Keywords: Annular Fin, Functionally Graded Material (FGM), Rotating Shaft

1. Introduction

The main purpose of an extended surface or a fin is to increase the rate of heat transfer from a heated surface to a cold fluid. Of these, annular fins find numerous applications in compact heat exchangers, specialized installations of single and double pipe heat exchangers, electrical apparatus with efficient heat dissipation, cylinders of air cooled internal-combustion engines and fuel cans in nuclear reactors to name a few. The design of a fin is considered to be optimum when the fins require minimum cost of manufacturing, offer the minimum resistance to the fluid flow, are light in weight and are easy to manufacture. A detailed review of literature on optimum design of fins has been carried out starting with [1] where upon using a set of idealizing assumptions, the efficiency of various straight fins and spines have been reported [2] solved the optimization problem of

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straight based fins assuming that the minimum weight fin had a linear temperature distribution along its length. In [3] presented an equation for the temperature gradient and the effectiveness of annular fins of constant thickness with a symmetrical temperature distribution around the base of the fin. In [4] reported the optimum dimensions of uniform annular fin by relating fin dimensions to the heat transfer and thermal properties of the fin and heat transfer coefficient between the fin and its surroundings.

The optimized dimensions of the fin can be found in either one of the two ways: the maximum amount of heat dissipation for a given quantity of weight or the minimum weight for dissipating a given quantity of heat. Ulmann and Kalman [5] adopted the first way and determined the efficiency and optimum dimensions of annular fins with triangular, parabolic, and hyperbolic profiles using numerical techniques. Dhar and Arora [6] described the methods of carrying out the minimum weight design of finned surfaces of specific type by first obtaining the optimum surface profile of a fin required to dissipate a certain amount of heat from the given surface, with no restriction on the fin height and then extended their study for the case when fin height is given. Duffin [7] gave a method for carrying out the minimum weight design of a fin using a rigorous mathematical method based on Variatioanal calculus and assumed constant thermal conductivity of a fin material and a constant heat transfer coefficient along the fin surface. In a recent work, Arauzo et al [8] reported a ten-term power series method for predicting the temperature distributions and the heat transfer rates of annular fins of hyperbolic profiles. Assuming fixed fin volume, Arslanturk [9] reported simple correlation equations for optimum design of annular fins with uniform cross sections to obtain the dimensionless geometrical parameters of the fin with maximum heat transfer rates. These simple correlation equations can help the thermal design engineers for carrying out the study on optimum design of annular fins of uniform thickness. In their recent work, Kundu and Das [10] reported the performance analysis and optimization of concentric annular fins with a step change in thickness using Lagrange multiplier.

Nomenclature

a, b	Grading parameter for thermal conductivity
C_1', C_1	Fin parameter
C_2', C_2	Parameter for rotation of fin
h	Convection heat transfer coefficient (W/m^2-K)
k_r	Thermal conductivity of the fin material at any radius (W/m-K)
L	Fin Length (m)
p,q	Coefficient for convective heat transfer coefficient
Q	Normalized Heat transfer
R_f	Aspect ratio, constant for fin shape relations (R_1/R_0)
r	Fin radius at the start of element (m)
R	Radius (m)
T	Temperature (^{0}C)
x	Dimensionless radial co-ordinate
Greek Symbols	
δ	Fin thickness (m)
δ_0	Fin thickness at the axis (m)
$\emptyset$	Dimensionless temperature
ω	Angular Velocity of shaft (rad/s)
Subscripts	
0	At the base of fin
1	At the tip of fin

In a recent work, Iborra and Campo [11] reported that approximate analytic temperature profiles and heat transfer rates of good quality are easily obtainable without resorting to the exact analytic temperature distribution and heat transfer rate based on modified Bessel functions. Kang [12] reported the optimum performance and fin length of a rectangular profile annular fin using variations separation method. Aziz and Fang [13] presented alternative solutions for different tip conditions of longitudinal fins having rectangular, trapezoidal and concave parabolic profiles and reported relationship between dimensionless heat flux, fin parameter and dimensionless tip temperature for all the geometries. Aziz and Khani [14] presented an analytical solution for thermal performance of annular fins of rectangular and different convex parabolic profiles mounted on a rotating shaft, losing heat by convection to its surroundings. In their work, convection heat transfer coefficient was assumed to be a function of radial coordinate and shaft speed.

In an experimental study, heat transfer rate and efficiency for circular and elliptical annular fins were analyzed for different environmental conditions by Nagarani[15]and high efficiency was reported for elliptical fins as compared to circular ones. An analytical solution for a rotating radial fin of rectangular profile were given by Aziz and Khani[16]. In this study, solution was obtained by Homotopy analysis Method (HAM) and compared with the direct numerical solution for both rectangular and parabolic profiles. In another study, Aziz and Fang [17] derived analytical expressions for the temperature distribution, tip heat flow and biot number at the tip and reported thermal performance of the annular fin under both cooling and heating conditions.

Moradi and Rafiee [18] solved the convection-radiation problem of moving longitudinal fin for varying profiles. Differential transformation method were applied for solving the governing equation and results indicate that exponential profile tip temperature is higher than the other profile.

In a recent work, Gaba et. al [19] reported the performance of functionally graded parabolic annular fin having constant weight where in the effect of grading and geometry parameter has been investigated and depends of fin performance on profile and grading is reported.

2. Mathematical Formulation

Assuming the effect of rotation on convective heat transfer coefficient being a linear function of angular velocity (ω) of shaft a relation is taken in the form of Aziz and Khani [14]

$$h=p+q.r.\omega \tag{1}$$

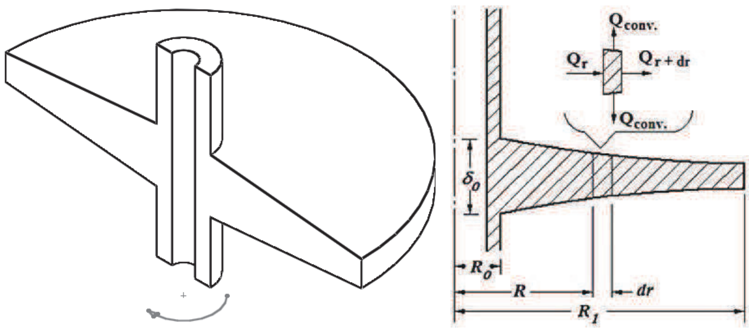

Fig. 1. Sectional isometric view of annular fin

The second order differential equation for the heat transfer through the fins has to be developed to determine the temperature profile. For calculating the heat balance, the details for a control volume of length '*dr*" of a fin is shown in Fig.1. This correlation equation (second order differential equation) has been solved using a computational algorithm. Thermal energy balance yields:

$$Q_r = Q_{r+dr} + Qconv. \tag{2}$$

Using Eqs. (2), the following equation can be arrived at

$$\frac{d}{dr}\left(k.A_r.\frac{d\theta}{dr}\right).dr = 4\pi r.h.\theta.dr.\sqrt{1+\frac{1}{4}.\left(\frac{d\delta}{dr}\right)^2}$$

The fin geometry and physical parameters are normalized using the following expressions,

$\phi = \frac{\theta}{\theta_0}$, $x = \frac{r - R_0}{R_1 - R_0} = \frac{r - R_0}{L}$, $R_f = \frac{R_1}{R_0}$ and $k_r = ar^b$

Upon introducing the normalized variables, the governing equation becomes,

$$\frac{d^2\phi}{dx^2} + A_1 \frac{d\phi}{dx} + A_2\phi = 0 \tag{3}$$

where,

$$A_1 = \frac{L}{Lx + R_0} + \frac{1}{k_r}\frac{dk_r}{dx} = \frac{L}{Lx + R_0} + \frac{Lb}{Lx + R_0} \tag{4}$$

$$A_2 = -\frac{(C_1^{'} + C_2^{'}x + R_0 C_2^{''})}{(Lx + R_0)^b} \tag{5}$$

where

$$C_1^{'} = \frac{2L^2 p}{a\delta_0}, C_2^{'} = \frac{2L^3 \omega q}{a\delta_0}, C_2^{''} = \frac{2L^2 \omega q R_0}{a\delta_0}$$

Eq. (3) can be generalized for varying thermal conductivity for rectangular profile. Equation (3) is solved using the following boundary conditions:

(i) $\phi = 1 \; at \; x = 0$

(ii) $\frac{d\phi}{dx} = 0 \; at \; x = 1$

Similarly for heat transfer; $q_{fin} = -\frac{2\pi a R_0^{b+1}\delta_0\theta_0}{L}\left(\frac{d\phi}{dx}\right)_{x=0}$ (6)

In dimensionless form; $Q = -\left(\frac{d\phi}{dx}\right)$ (7)

The dimensionless temperature ϕ depends on ***C₁'***, ***C₂'*** and ***C₂''***. ***C₁'*** is the fin parameter and *C₂'* and ***C₂''*** another parameter that involve the rotation of the fin. Further C_1 and C_2 can be defined as follows:

$$C_1 = \frac{C_1^{'}}{(1 - R_f^{-1})^2} \text{ and } C_2 = \frac{C_2^{'}}{(1 - R_f^{-1})^2}$$

Consider x as the only independent variable, the Eq. (3), are solved for different values of grading parameter ***b*** based on a range of C_1 and C_2.

3. Result and Discussion

The temperature distribution and heat transfer of the fin is calculated by evaluating the first derivative of temperature at the fin base i.e. $\phi'_{x=1}$ by solving second order differential equation in MATLAB using subroutine bvp4c Shampine et. Al [19]. Subroutine bvp4c is a proven technique to solve boundary value problems by choosing a continuous piecewise polynomial that adjusts to the boundary conditions. The remaining unknown coefficients are determined by collocating the algebraic equations at several points. It is an adaptive finite difference code that employs three-stage Lobatto III-a collocation formula by computing a cubic spline on each subinterval[x_i, x_{i+1}] of a mesh. The approximate continuous solution is obtained by controlling the residual over each subinterval [x_i, x_{i+1}] approximated using a five-point Lobatto quadrature formula.

Literature survey points towards the nonexistence of studies addressing the problem of functionally-graded rotating annular fins. Validation of the formulation is presented for annular fin of rectangular profile considering the isotropy of material Aziz and Khani [14] and presented in Fig. 2. The results are found to be in good agreement with benchmark results. This is followed by a parametric study for a grading parameter i.e. ***b***=-1 for rectangular profile. The results obtained for different rotational speed for assumed grading parameter are presented in the following graphs and discussed.

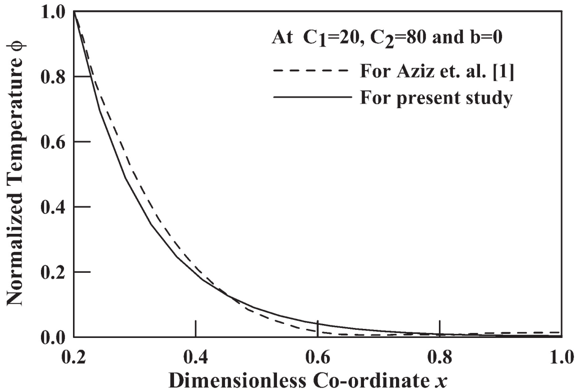

Fig. 2. Distribution of Normalized Temperature over the Dimensionless co-ordinate.

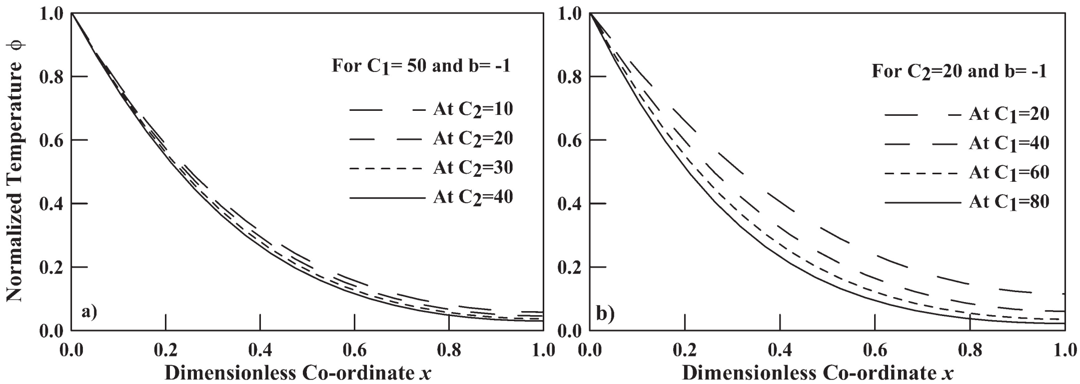

Fig. 3 Normalized Temperature distribution along with the fin radius

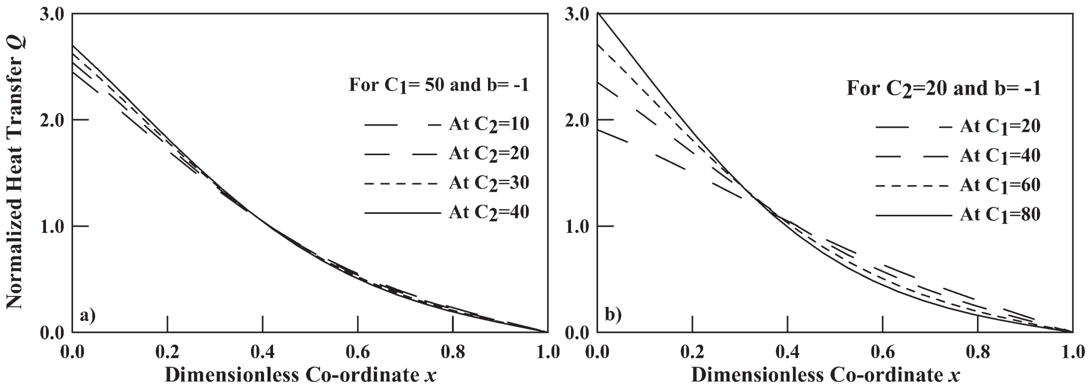

Fig. 4 Normalized Heat Transfer along with the fin radius

For rectangular profile, the effect of parameter C_1 and C_2 on the temperature distribution along the radius is shown in Fig. 3. It is observed from Fig. 3 that temperature at the tip decreases with increase in C_2 at the grading parameter b =-1. Similar observation is also reported for the increase in C_1 for any C_2, but influence of fin parameter C_1 is more on temperature distribution along the radius compared to parameter C_2. This reveals that higher the value of fin parameter C_1 and rotating parameter C_2, sharper the decline in temperature profile thus indicating that the portion near the tip of the fin is not participating in heat transfer. As a result, the performance of the fin reduces with the increase in fin parameter and rotating parameter for the same aspect ratio. However, this also provides the opportunity to reduce the size of the fin and improve the performance of the fin for a reduced size.The performance of fin is also analysed by calculating the normalized heat transfer by varying the values of fin parameter and rotating parameter and are depicted in Fig. 4. It is evident from the figure that higher values $\boldsymbol{C_1}$ and C_2 along with high thermal conductivity near the base ($\boldsymbol{b}$ = -1) gives higher heat transfer. The heat transfer reduces sharply towards the tip as excess temperature is small due to the combined effect of grading and rotation of the fin. This may be attributed to the fact that the temperature distribution (shown in fig. 3) provides higher heat transfer. It is evident that at the base, for higher values of $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$, heat transfer is higher but it decreases sharply to become least at the tip. This is due to increase in convective heat transfer coefficient with increase is $\boldsymbol{C_2}$.

4. Conclusion

The performance of functionally graded annular fins having rectangular profile subjected to rotation is reported. The study is carried out for different value of parameter $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$ and grading parameter $\boldsymbol{b}$. It is observed that for increase in fin parameter $\boldsymbol{C_1}$ and rotational speed parameter $\boldsymbol{C_2}$ the performance of the fin decreases for the same aspect ratio of the fin, but the size of fin can be reduced and performance of the fin is increased for the reduced aspect ratio. The study of effect of grading parameter on fin performance indicates that both temperature distribution and heat transfer are enhanced with decrease in grading parameter.

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